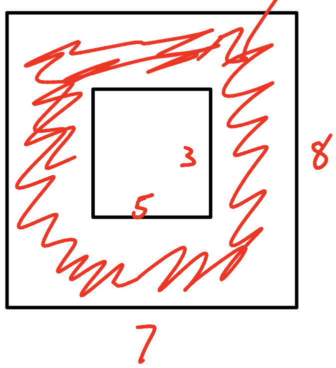
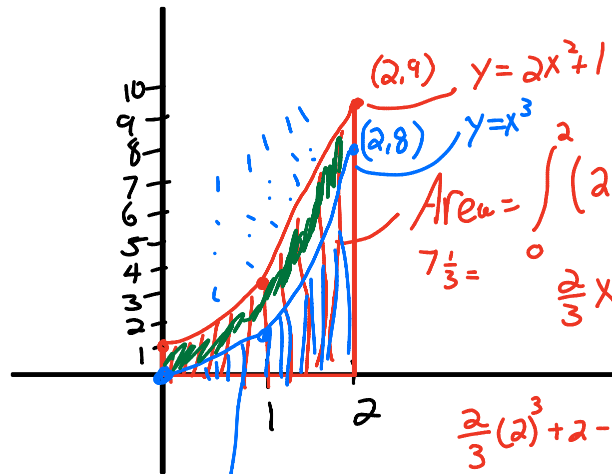
Period 3, Feb 26, 2025

$$\text{Area} = 8\cdot 7 - 5\cdot 3 = 56 - 15 = 41$$





$$\text{Area} = \int_0^2 (2x^2+1)\,dx$$

$$7\tfrac{1}{3} = \quad \frac{2}{3}x^3 + x \Big[_0^2$$

$$\frac{2}{3}(2)^3 + 2 - \cancel{\left[\frac{2}{3}(0)^3 + 0\right]}$$

$$\frac{16}{3} + 2 = 5\tfrac{1}{3} + 2 = 7\tfrac{1}{3}$$

$F(x) = 2x^2 + 1$

$g(x) = x^3$

$$\int_0^2 x^3\,dx = \frac{1}{4}x^4\Big[_0^2 = \frac{16}{4} - \frac{0}{4} = 4$$

$$\text{Area} = 7\tfrac{1}{3} - 4 = 3\tfrac{1}{3} = \frac{10}{3}$$

$$\int_0^2 (2x^2+1)\,dx - \int_0^2 x^3\,dx$$

$$\int_0^2 \left[(2x^2+1) - (x^3)\right] dx$$

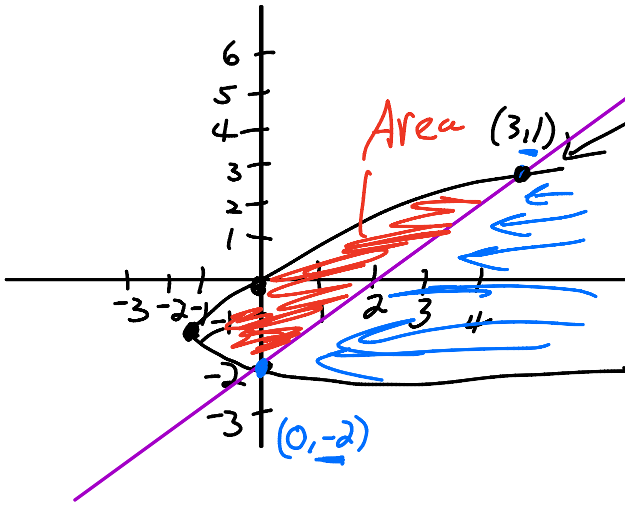
if Rain Falls Down

First To get Hit by Rain

second To get Hit by Rain

How does Rain Fall? = Down

From + To −



$x=(y+1)^2-1$

$1+2=3$

$y=\ x-2 \Rightarrow y+2=x$

Rain Falls From Right To Left

$\int\left((x-2)-(y+1)^2-1\right)dy$

if Rain Falls Sideways

$y=x-2 \Rightarrow y+2=x$

$x=(y+1)^2-1$

$y+2=(y+1)^2-1$

$y+2=y^2+2y+\cancel{1-1}$

$-y\ -2 \qquad -y\ -2$

$0=y^2+y-2$

$0=(y+2)(y-1)$

$y+2=0$ or $y-1=0$

$y=-2 \quad y=1$

$\int_{-2}^{1}\left[(y+2)-\left((y+1)^2-1\right)\right]dy$

$\int_{-2}^{1}\left[y+2-(y^2+2y+\cancel{1-1})\right]dx$

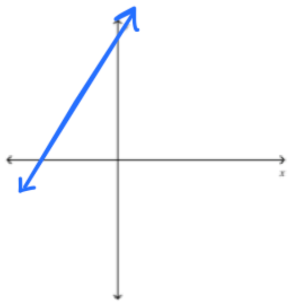
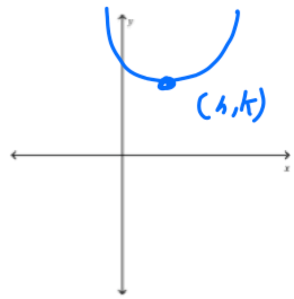
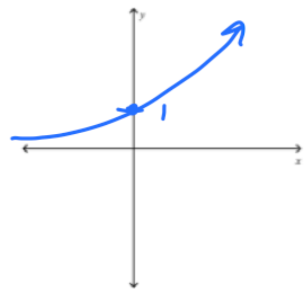
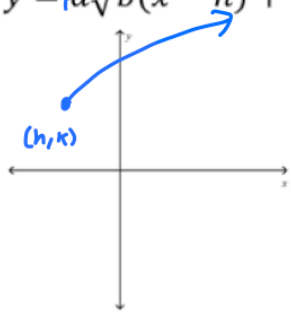
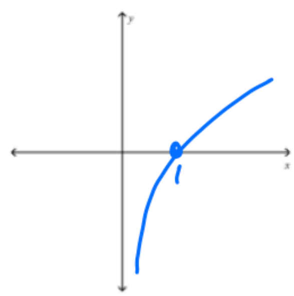
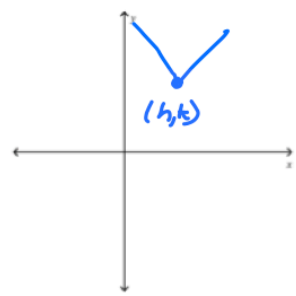
$\int_{-2}^{1}\left[y+2-y^2-2y\right]dy=\int_{-2}^{1}\left[-y^2-y+2\right]dy$

$-\frac{1}{3}y^3-\frac{1}{2}y^2+2y\Big[_{-2}^{1}$

$\left[-\frac{1}{3}(1)^3-\frac{1}{2}(1)^2+2(1)\right]-\left[-\frac{1}{3}(-2)^3-\frac{1}{2}(-2)^2+2(-2)\right]$

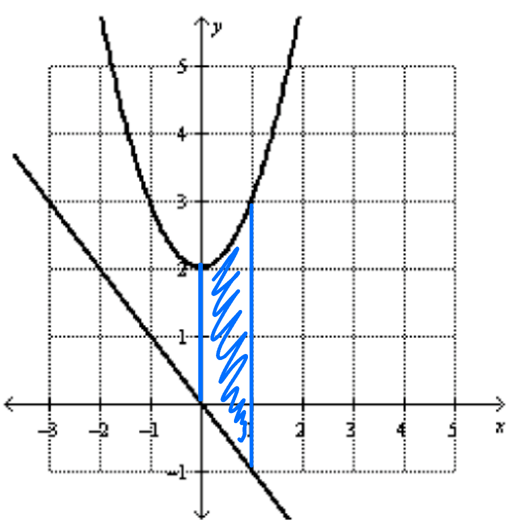
$-\frac{1}{3}-\frac{1}{2}+2-\frac{8}{3}+2+4=-\frac{9}{3}-\frac{1}{2}+8=-3-\frac{1}{2}+8=5-\frac{1}{2}=4\frac{1}{2}$

Sketching Common Graphs

$y = mx + b$	$y = \pm a(x-h)^2 + k$	$y = e^x$
		
$y = \pm a\sqrt{b(x-h)} + k$	$y = \ln x$	$y = \pm a\lvert x-h\rvert + k$
		

Example 1

Find the area of the graph of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$



$$\int_0^1 (x^2+2)\,dx - \int_0^1 -x\,dx$$

$F(x) = x^2+2$

$$\int_0^1 [(x^2+2) - (-x)]\,dx = \int [F(x) - g(x)]\,dx$$

$g(x) = -x$

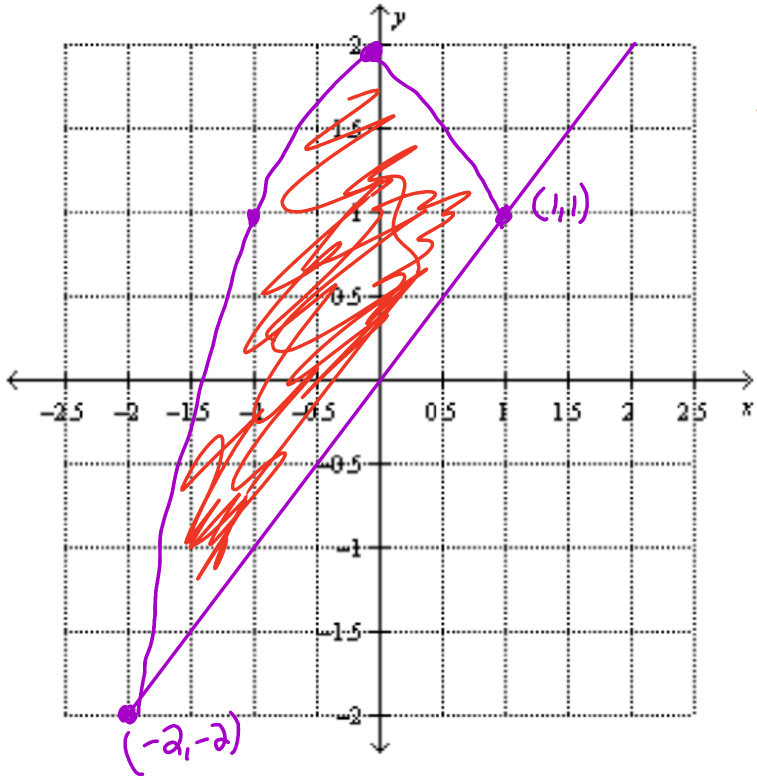
$$\int_0^1 (x^2+2+x)\,dx = \frac{1}{3}x^3 + 2x + \frac{1}{2}x^2 \Big|_0^1$$

$$\frac{1}{3}(1)^3 + 2(1) + \frac{1}{2}(1)^2 - \left[\cancel{\frac{1}{3}(0)^3 + 2(0) + \frac{1}{2}(0)^2}\right]$$

$$\frac{2\cdot 1}{2\cdot 3} + 2 + \frac{1\cdot 3}{2\cdot 3} = \frac{2}{6} + 2 + \frac{3}{6} = 2 + \frac{5}{6} = 2\frac{5}{6}$$

Example 2

Find the area of the graph of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.



$x = 2 - x^2$

$+x^2 - 2 \quad -2 \quad +x^2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x+2=0$ or $x-1=0$

$x=-2 \qquad x=1$

$\int_{-2}^{1} [(2-x^2)-(x)]\,dx$

$\int_{-2}^{1} [2-x^2-x]\,dx$

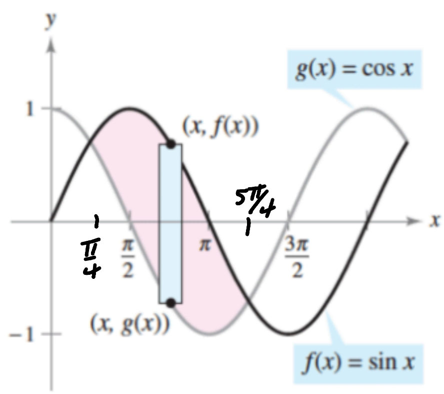
$2x - \frac{1}{3}x^3 - \frac{x^2}{2} \Big[_{-2}^{1}$

$\left[2(1) - \frac{1}{3}(1)^3 - \frac{(1)^2}{2}\right] - \left[2(-2) - \frac{1}{3}(-2)^3 - \frac{(-2)^2}{2}\right]$

$2 - \frac{1}{3} - \frac{1}{2} + 4 - \frac{8}{3} + 2 = 8 - \frac{9}{3} - \frac{1}{2} = 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = 4\frac{1}{2}$

Example 3

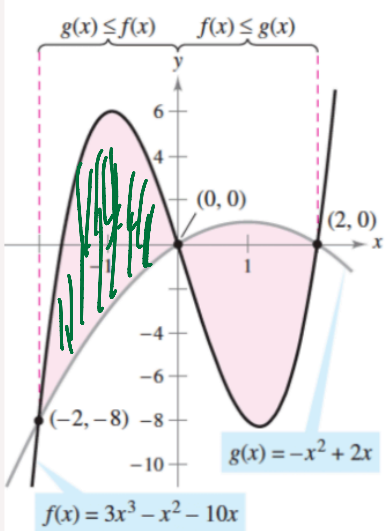
The sine and cosine curves interest infinitely many times, bounded regions of equal areas. Find the area of one of these regions.



$$\int_{\frac{\pi}{4}}^{5\pi/4} [\sin x - \cos x]\, dx = -\cos x - \sin x \Big[_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

Example 5

Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$



$$\int_{-2}^{0} [F(x) - g(x)]dx + \int_{0}^{2} [g(x) - F(x)]dx$$

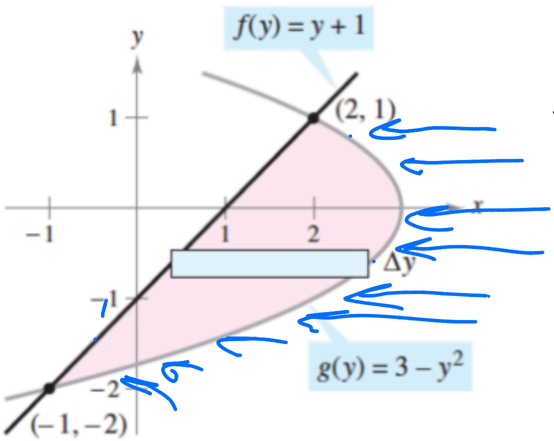
$$\int_{-2}^{0} [(3x^3 - x^2 - 10x) - (-x^2 + 2x)]dx + \int_{0}^{2} [(-x^2 + 2x) - (3x^3 - x^2 - 10x)]\, dx$$

$$\int_{-2}^{0} (3x^3 + 0 - 12x)dx + \int_{0}^{2} (12x - 3x^3)dx$$

$$\left(\frac{3}{4}x^4 - 6x^2\right)\Big[_{-2}^{0} + \left(6x^2 - \frac{3}{4}x^4\right)\Big[_{0}^{2}$$

Example 6 (cont)

Find the area of the region between the graphs of $x = 3 - y^2$ and $x = y + 1$

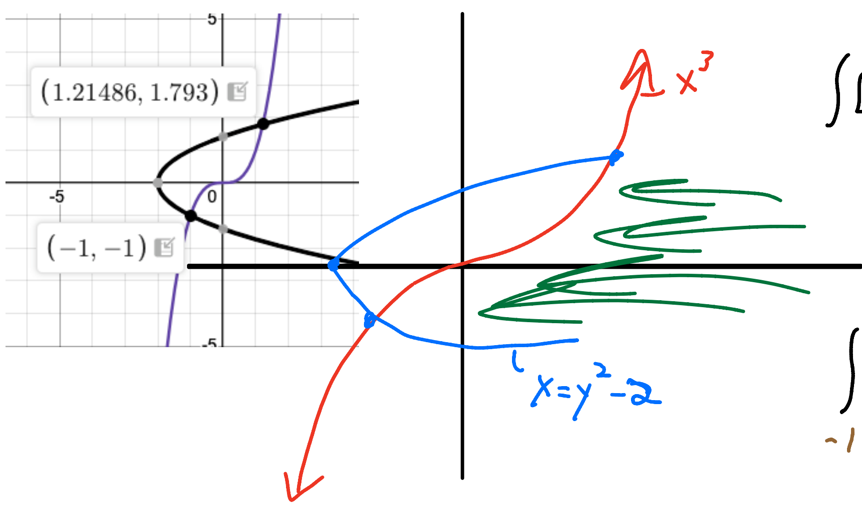


$$\int_{-2}^{1} [(3-y^2)-(y+1)]\,dy$$

Example 7 (cont.)

$\sqrt[3]{y} = y^{\frac{1}{3}} = x$

Example 7 Find the area of the region enclosed by the graphs of $y = x^3$ and $x = y^2 - 2$.



$$\int [x^3-(y^2-2)]\,dy$$

dy

$$\int_{-1}^{1.793} [y^{\frac{1}{3}} - (y^2-2)]\,dy$$

$y = x^3$ and $x = y^2 - 2$